CBCS SCHEME

USN 18CS36

Third Semester B.E. Degree Examination, July/August 2022
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define Tautology. Prove that for any propositions p, q, r the compound proposition $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. (06 Marks)

b. Test the validity of the arguments using rules of inference.

$$\begin{array}{l} (\neg p \vee q) \to r \\ r \to s \vee t \\ \neg s \wedge \neg u \\ \neg u \to \neg t \\ \hline \therefore p \end{array}$$

(06 Marks)

c. Give an indirect proof and proof by contradiction for, "If m is an even integer, then m + 7 is odd". (08 Marks)

OR

2 a. Prove the following logical equivalences using laws of logic:
$[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ (06 Marks)

b. Consider the following open statements with the set of all real numbers as the universe:
$p(x): x \geq 0$, $q(x): x^2 \geq 0$, $r(x): x^2 - 3x - 4 = 0$
$s(x): x^2 - 3 > 0$. Determine the truth values of the following statements.

(i) $\exists x, p(x) \wedge q(x)$
(ii) $\forall x, p(x) \to q(x)$
(iii) $\forall x, q(x) \to s(x)$
(iv) $\forall x, r(x) \vee s(x)$
(v) $\exists x, p(x) \wedge r(x)$
(vi) $\forall x, r(x) \to p(x)$ (06 Marks)

c. Establish the validity of the following :

$$\begin{array}{l} \forall x, [p(x) \vee q(x)] \\ \exists x, \neg p(x) \\ \forall x, [\neg q(x) \vee r(x)] \\ \forall x, [s(x) \to \neg r(x)] \\ \hline \therefore \exists x, \neg s(x) \end{array}$$

(08 Marks)

Module-2

3 a. Prove by mathematical induction $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. (06 Marks)

b. A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting atleast 2 questions from each part? (06 Marks)

c. Determine the coefficient of,
(i) xyz^2 in $(2x - y - z)^4$ (ii) x^9y^3 in the expansion of $(2x - 3y)^{12}$. (08 Marks)

OR

4 a. Prove by mathematical induction, $1.3 + 2.4 + 3.5 + ... + n(n+2) = \frac{n(n+1)(2n+7)}{6}$. **(06 Marks)**

b. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these all four A's are together? How many of them begin with S? **(06 Marks)**

c. In how many ways can we distribute eight identical white balls into four distinct containers so that,

(i) no container is left empty?

(ii) the fourth container has an odd number of balls in it? **(08 Marks)**

Module-3

5 a. State pigeonhole principle. ABC is an equilateral triangle whose sides are of length 1 cm each. If we select 5 points inside the triangle, prove that atleast two of these points are such that the distance between them is less than $\frac{1}{2}$ cm. **(08 Marks)**

b. If $A = A_1 \cup A_2 \cup A_3$ where $A_1 = \{1, 2\}$, $A_2 = \{2, 3, 4\}$ and $A_3 = \{5\}$, define a relation R on A by xRy if x and y are in the same subset A_i for $1 \le i \le 3$. Is R an equivalence relation. **(06 Marks)**

c. Let $f, g : R \to R$ where $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(gof)(x) = 9x^2 - 9x + 3$ determine a, b. **(06 Marks)**

OR

6 a. Prove that if $f : A \to B$, $g : B \to C$ are invertible functions, then $gof : A \to C$ in invertible and $(gof)^{-1} = f^{-1} o g^{-1}$. **(06 Marks)**

b. For $A = \{a, b, c, d, e\}$ the Hasse diagram for the poset (A, R) is shown in Fig. Q6 (b).

(i) Determine the relation matrix for R.

(ii) Construct the directed graph G that is associated with R. **(06 Marks)**

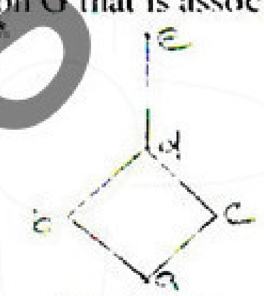

Fig. Q6 (b)

c. If R is an equivalence relation on a set A and $x, y \in A$ then prove

(i) $x \in [x]$

(ii) xRy if and only if $[x] = [y]$ and

(iii) if $[x] \cap [y] \neq \varphi$ then $[x] = [y]$. **(08 Marks)**

Module-4

7 a. Find the number of permutations of a, b, c, ..., x, y, z in which none of the patterns spin, game, path or net occurs **(08 Marks)**

b. For the positive integers 1, 2, 3, ...n there are 11660 derangements where 1, 2, 3, 4 and 5 appear in the first five positions. What is the value of n? **(06 Marks)**

c. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ where $n \ge 2$ and $a_0 = -1$, $a_1 = 8$. **(06 Marks)**

OR

8 a. Determine the number of integers between 1 and 300 (inclusive) which are, (i) divisible by exactly two of 5, 6, 8 …. (ii) divisible by atleast two of 5, 6, 8. **(06 Marks)**

b. Describe the expansion formula for Rook polynomials. Find the Rook polynomial for 3×3 board using expansion formula. **(08 Marks)**

c. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(06 Marks)**

Module-5

9 a. Define with examples. (i) Subgraph, (ii) Spanning subgraph (iii) Complete graph (iv) Induced subgraph (v) Complement of a graph (vi) path. **(06 Marks)**

b. Merge sort the list.
$-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3$ **(06 Marks)**

c. Define isomorphism of two graphs. Determine whether the following graphs G_1 and G_2 are isomorphic or not.

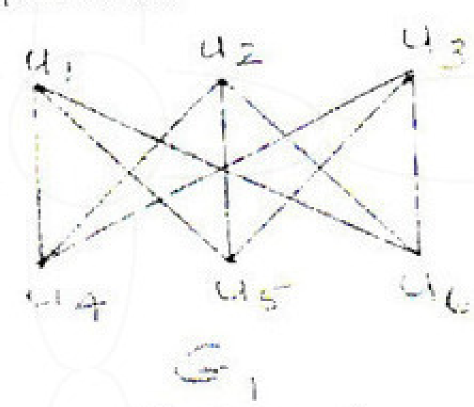

Fig. Q9 (c) – i

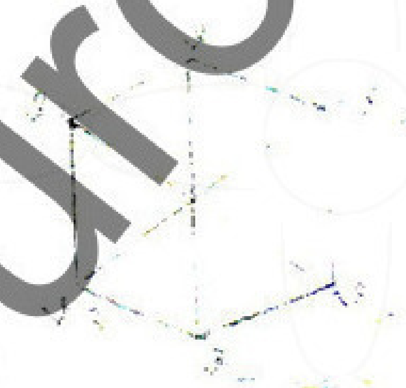

Fig. Q9 (c) - ii

(08 Marks)

OR

10 a. Let G = (V, E) be the undirected graph in Fig. Q10 (a). How many paths are there in G from a to h? How many of these paths have length 5? **(06 Marks)**

Fig. Q10 (a)

b. Prove that in every tree T = (V, E), $|V| = |E| + 1$ **(06 Marks)**

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(08 Marks)**

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